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THE STOCHASTIC FORMULATION OF A MODIFIED COBWEB MODEL

by

Barry D. Nussbaum
Nozer D. Singpurwalla

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Much of the economic analysis done today employs either econometric models or autoprojective methods on series of data. In this paper we examine the connection between these two methods, and demonstrate how formulating an econometric model as a stochastic process can be useful. The basic economic cobweb model for price and quantity is used to generate an autoregressive process for the price series. Analyses are made of the speed with which price equilibrium is achieved. The familiar stability conditions are compared to the stationarity and invertibility conditions of a time series process. Forecast functions are derived, and an example is given to demonstrate how data can be tested to see if the underlying econometric mechanism is relevant.

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1. Introduction

Recently, several papers comparing the predictive performance of complex econometric models with those based on simple autoprojective methods have appeared in the literature, [Nelson (1973), Cooper and Nelson (1975), Cooper and Jorgenson (1969), Narasimham, Castellino, and Singpurwalla (1974)]. Cooper and Jorgenson (1969) conclude that "econometric models are not in general superior to purely mechanical (autoprojective) models for forecasting." An understandable criticism levied against such autoprojective methods is that they lack an economic basis. It is thus apparent that the need to interpret an autoprojective model in the light of econometric theory is quite germane to econometric analysis. A recent endeavor in bridging this gap between economic and autoprojective models is the excellent work of Zellner and Palm (1974).

In this paper, we attempt to undertake an endeavor similar in spirit to that of Zellner and Palm. However, we shall focus attention on a modified version of the simple cobweb model. Our reasons for choosing this model stems from the fact that i) it is basic to econometric theory [Nicholson (1972)], and ii) the data presented in Section 4 of this paper lend credence to its validity. In Section 4 we present some data

on the price of cotton in the U.S. between 1929 and 1971. Using the Box-Jenkins (1970) technique of time series analysis, we claim that an autoregressive process of order one AR(1), is adequate to describe the price series. In Section 3 we argue that our claimed model is valid if we are willing to assume that a modified version of the cobweb model is the generating mechanism of the price series.

Another objective of this paper, is to point out the fact that some econometric models can also be analyzed by formulating them as a discrete time stochastic process. Specifically, such a formulation enables us to

a) interpret the stability condition of economic equilibrium in the light of the stationarity and the invertibility conditions of a stochastic process,

b) investigate the speed with which economic equilibrium is attained, and

c) obtain expressions for the forecast function for the variable of interest (price, in our case).

In addition to the above, a stochastic (time series) process formulation provides us with a statistical mechanism for testing the appropriateness of a hypothesized model.

In Section 2 of our paper, we present a brief introduction to the cobweb model, and a minor variation of it. This model is an example of short-run supply-demand dynamics. In econometric theory, the model is presented as a deterministic one, with certain values of the parameters leading to a stable equilibrium for the price, whereas other values lead to an explosive series for the price.

In Section 3, we formulate the cobweb model and its variation as a discrete time stochastic process by imposing a simple error structure on it. In Section 4 we present a time series analysis of some data given in Appendix A; the analysis of these data motivates our choice of the cobweb model and its variation.

2. The Cobweb Model

The basic version of a cobweb model is often attributed to the agricultural sector of a competitive economy. Briefly, the model states that farmers generate the quantity of a product to be supplied in the next period as a function of this period's market price. That is,

$$Q_t^S = \alpha + \beta\, P_{t-1} \tag{2.1}$$

where Q_t^S = quantity supplied in period t

P_t = market price in period t, and α and β are unknown coefficients.

Demanders of the commodity, on the other hand, determine the quantity desired at time t, Q_t^D as a function of the current (time t) price as

$$Q_t^D = \gamma + \delta P_t \tag{2.2}$$

where γ and δ are another set of unknown coefficients. By economic considerations, the signs of α, β, and γ are assumed to be positive. The sign of δ is assumed to be negative.

Equations (2.1) and (2.2) represent a behavioral relationship between quantity and price, and is assumed to exist in a competitive economy. To be able to develop a deterministic system of equations, it is necessary to impose an identity called a "clearing equation," which equates the quantities supplied and demanded at time t. Thus,

$$Q_t^S = Q_t^D\ . \tag{2.3}$$

From a behavioral point of view, it is reasonable to conjecture that the above relationship holds only after some decisions on the part of suppliers and demanders have taken place. However, in keeping with the spirit of presenting a simple model with which to demonstrate these ideas, Equation (2.3) will be assumed true for all time periods.

Based upon the above, a basic conclusion from the cobweb model is that, if $|\beta| < |\delta|$, $\lim\limits_{t\to\infty} P_t \overset{\text{def}}{=} P^*$, and thus P_t achieves what is known

as a price equilibrium. If, however, $|\beta| > |\delta|$, then P_t explodes, whereas $|\beta| = |\delta|$, P_t perpetually oscillates about some point [see e.g., Nicholson (1972)].

Several authors, particularly Akerman (1957) and Rader (1972) have questioned the elementary and primitive nature of this model. In response to such criticism, Nerlove (1958) has proposed modifications to strengthen the plausibility of the model. Nerlove's modification, called the adaptive expectations model is of significant interest and raises several technical issues; these will be discussed in a forthcoming paper.

In this paper, we introduce a (minor) variant of the basic cobweb model and compare it with the original model. This variant regards quantity decisions as a function of the change in price between successive periods. Specifically, a supplier always supplies a fixed quantity α , and adds on to it a quantity proportional to the change in price between the period $t-1$ and $t-2$. Thus,

$$Q_t^S = \alpha + \beta(P_{t-1}-P_{t-2}) \ . \tag{2.4}$$

Thus, if price has recently increased $P_{t-1} > P_{t-2}$, the supplier will tend to supply greater quantities (β is assumed to be positive). Similarly, the demander sets his demand schedule based on the change in prices as follows:

$$Q_t^D = \gamma + \delta(P_t-P_{t-1}) \tag{2.5}$$

with δ assumed to be negative. The clearing Equation (2.3) is still presumed to hold.

We note that the modified version is essentially equivalent to the original cobweb model, with $\Delta P_t = P_t - P_{t-1}$ replacing P_t and ΔP_{t-1} replacing P_{t-1} . Thus, all the results and conclusions which pertain to the cobweb model carry over to its modified version, with ΔP_t in place of P_t , for all values of t .

In the next section, we shall formulate the simple cobweb model as a discrete time stochastic process. The stationarity conditions of the

process will throw some light on the behavior of the price structure, whereas the invertibility conditions will give us some insight into the "memory" of the price series. The forecast function of the process will enable us to predict individual values of the future prices. In summary, the well-developed theory of stochastic processes and time series analysis will be used to analyze the behavior of the price series.

3. The Cobweb Model as a Discrete Time Stochastic Process

It is reasonable to assume that random disturbance components u_t and ν_t will be associated with Q_t^S and Q_t^D, respectively. Thus, we rewrite Equations (2.1) and (2.2) as

$$Q_t^S = \alpha + \beta P_{t-1} + u_t \tag{3.1}$$

and

$$Q_t^D = \gamma + \delta P_t + \nu_t , \qquad t = 0,1,2,... \tag{3.2}$$

3.1 The Error Theory Assumptions

In order to formalize the model structure, and, in order to develop a mechanism for testing the validity of the model, we shall make some assumptions about u_t and ν_t.

i) u_t and ν_t, are both normally distributed with means zero, and variances σ_u^2 and σ_ν^2, respectively,

ii) $E(u_t u_{t-j}) = E(\nu_t \nu_{t-j}) = 0$, for all t and $j > 0$; thus, the individual disturbances are independent.

iii) $E(u_t \nu_{t-j}) = 0$, for all t and j; thus the u_t and the ν_t are mutually independent.

Comment

We remark here that the above assumptions are the simplest ones that we can make about the random disturbances. It is under these assumptions that the known results about the cobweb model can be interpreted.

If the above assumptions are changed (strengthened), the conclusions from the model will be different from those given in Section 2. The error theory also provides us with a vehicle for enlarging the scope of this model. This is because a different set of assumptions will lead us to a different set of models for the underlying stochastic process, and these may be more appropriate for explaining some real life data.

3.2 The Cobweb Model as an Autoregressive Process

Following our discussion in Section 2, we shall assume that the clearing equation holds, and that for large values of t

$$\alpha + \beta P_{t-1} + u_t = \gamma + \delta P_t + \nu_t$$

giving us

$$P_t = \frac{\alpha-\gamma}{\delta} + \frac{\beta}{\delta} P_{t-1} + \frac{u_t - \nu_t}{\delta} . \tag{3.3}$$

Equation (3.3) is the reduced form equation for price.

Under the assumptions of Section 3.1, $(u_t - \nu_t)/\delta$ is also a random disturbance, and thus P_t is described by an autoregressive process of order one. This process is always invertible, and is stationary if $\left|\frac{\beta}{\delta}\right| < 1$ [cf. Box and Jenkins (1970)]. If $|\beta| > |\delta|$, or if $|\beta| = |\delta|$, the process is non-stationary. The condition for stationarity is identical to the condition for price equilibrium (stability) stated in Section 2. Thus, when the process is stationary, we say that price equilibrium is attained, in which case, for t large $E(P_t) = E(P_{t-1}) = E(P^*)$. The error theory assumptions together with Equation (3.3) immediately imply that

$$E(P^*) = \frac{\alpha-\gamma}{\delta-\beta} .$$

When $|\beta| > |\delta|$, the process is not stationary, and, also, price equilibrium is not achieved since P_t is known to explode. In order to investigate the nature of this non-stationarity, that is, to see if non-stationarity is caused by a change in the mean, or a change in the variance,

we shall look at a moving average representation of this process. This will give us a better insight into the nature of the non-stationarity and this will be done in the following section.

When $|\beta| = |\gamma|$, the autoregressive representation leads us to an interesting observation. We set $w_t = (u_t - \nu_t)/\delta$ and note that the w_t are normally distributed with a mean of zero and a variance of $(\sigma_u^2+\sigma_\nu^2)/\delta^2$. It is now easy to verify that the second difference of P_t (i.e., $P_t-2P_{t-1}+P_{t-2}$) is a moving average process of order one and this is always stationary. Specifically, if ∇ denotes a difference operator, then

$$\nabla^2 P_t = w_t - w_{t-1}$$

where $w_t - w_{t-1}$ is a non-invertible moving average process.

If the stationarity of a process is to be identified with price equilibrium, then, based on the above, we can conclude that for $|\beta| = |\delta|$, the second difference of the price attains an equilibrium.

3.3. The Cobweb Model as a Moving Average Process

A moving average form of the cobweb model is more interesting than its autoregressive counterpart. This is because for $|\beta| \geq |\delta|$, the moving average form enables us to investigate the nature of the non-stationarity of the P_t process.

We shall denote the initial price by P_0, and set $\frac{\alpha-\gamma}{\delta} = \mu$. If the ratio β/δ is denoted by ϕ_1, then Equation (3.3) can be written as

$$P_t = \mu + \phi_1 P_{t-1} + w_t .$$

It is easy to verify that, in terms of P_0, P_t can also be written as

$$P_t = \phi_1^t\left(P_0 - \frac{\mu}{1-\phi_1}\right) + \frac{\mu}{1-\phi_1} + \phi(w)$$

where

$$\phi(w) = w_t + \phi_1 w_{t-1} + \phi_1^2 w_{t-2} + \ldots + \phi_1^{t-1} w_1 .$$

To simplify our notation we introduce the backward shift operator $B^m w_t = w_{t-m}$, $m = 0, 1, 2, \ldots$. Then, $\phi(w) = (1+\phi_1 B+\phi_1^2 B^2+\ldots+\phi_1^{t-1} B^{t-1}) w_t$

$$= \phi_{t-1}(B) w_t ,$$

where $\phi_{t-1}(B)$ is a <u>moving average filter</u> of order $(t-1)$.

In terms of our original variables in Equation (3.3), moving average form reduces to

$$P_t = \left(\frac{\beta}{\delta}\right)^t [P_0 - E(P^*)] + E(P^*) + \phi_{t-1}(B) w_t . \tag{3.4}$$

We shall now study this model for different conditions on β and δ and for different values of t .

<u>Case 1</u>. $\beta = -\delta \Rightarrow \phi_1 = -1$.

Under this case, we note that

$$P_t = (-1)^t [P_0 - E(P^*)] + E(P^*) + \phi_{t-1}(B) w_t$$

where

$$E(P^*) = (\alpha-\gamma)/2\delta \text{ , and}$$

$$\phi_{t-1}(B) w_t = \begin{cases} (1-B+B^2-\ldots+B^{t-1}) w_t \text{ , if } t \text{ is odd.} \\ (1-B+B^2-\ldots-B^{t-1}) w_t \text{ , if } t \text{ is even.} \end{cases}$$

For every value of t , $E\left[\phi_{t-1}(B) w_t\right] = 0$, and thus,

$$E(P_t) = \begin{cases} P_0 & \text{, if } t \text{ is even} \\ \frac{\alpha-\gamma}{\delta} - P_0 & \text{, if } t \text{ is odd.} \end{cases}$$

Since the expected value of P_t oscillates between P_0 and $\frac{\alpha-\gamma}{\delta} - P_0$, depending on the sign of t, the process P_t is not stationary and thus price equilibrium is not attained.

However, the variance of P_t can be seen from Equation (3.4) to be the variance of $\phi_{t-1}(B)w_t$. Thus,

$$\begin{aligned} Var(P_t) &= Var[\phi_{t-1}(B)w_t] \\ &= Var[1-B+B^2-\ldots+B^{t-1}]w_t \\ &= t\sigma^2 , \end{aligned}$$

for finite values of t, $t\sigma^2$ is finite and hence the P_t process has a finite variance.

If we define $\tilde{P}_t = P_t - E(P_t)$, then

$$\tilde{P}_t = \phi_{t-1}(B)w_t$$

is, for any <u>finite</u> value of t, a moving average process of finite order and, thus, the process $\tilde{P}_t$ is always stationary.

Based on the above, we conclude that for $\beta = -\delta$, the process P_t is non-stationary and that the non-stationarity is due to a fluctuating mean rather than a change in the higher order moments (e.g., variance) of the process. Of course, we recall that the second difference for P_t is a stationary process (see Section 3.2).

<u>Case 2</u>. $|\beta| > |\delta| \Rightarrow |\phi_1| > 1$.

Under this case, we note that

$$P_t = \phi_1^t[P_0-E(P^*)] + E(P^*) + \phi_{t-1}(B)w_t ,$$

and

$$E(P_t) = \phi_1^t[P_0-E(P^*)] + E(P^*) .$$

Since β and δ have opposite signs, and since ϕ_1^t increases in t, $E(P_t)$ will fluctuate in sign while increasing in t.

Here again, $\tilde{P}_t = P_t - E(P_t)$ is for finite values of t a moving average process of finite order which is always stationary.

Here again, we conclude that for $|\beta| > |\delta|$, the process P_t is non-stationary in the mean.

Case 3. $|\beta| < |\delta| \Rightarrow |\phi_1| < 1$.

This case, we recall, was the one which gave us price equilibrium, and stationarity when P_t was written as an autoregressive process. Out of interest, we shall now see how a moving average representation of this process will also lead to the same conclusion. We first note that in

$$P_t = \phi_1^t [P_0 - E(P^*)] + E(P^*) + \phi_{t-1}(B)w_t ,$$

ϕ_1^t decreases in t such that

$$\lim_{t\to\infty} E(P_t) = E(P^*) .$$

Thus, after some initial fluctuations, the expected value of the process converges to $E(P^*) = (\alpha-\gamma)/(\delta-B)$, a constant.

The above observation proves that the P_t process is stationary in the mean (weakly stationary) for large values of t. In order to prove (strong) stationarity of the P_t process, we will have to examine the moving average filter,

$$\phi_{t-1}(B) = (1+\phi_1 B+\phi_1^2 B^2+\ldots+\phi_1^{t-1} B^{t-1}) .$$

It is well known that a finite moving average process is always stationary. However, in our case, for large values of t, the number of moving average terms given by $\phi_{t-1}(B)$ increases indefinitely. Thus, in order to argue that P_t is stationary it is necessary to be assured that for large values of t the number of terms in $\phi_{t-1}(B)$ is essentially finite. Since the coefficients of the B's in the moving average filter are powers of ϕ_1, it suffices to note that for large values of t, $\phi_1^t \approx 0$.

3.3.1 <u>Time at Which Price Equilibrium is Achieved</u>. Since price equilibrium is achieved when the process becomes strictly stationary, the time at which this occurs is of interest. However, the time at which stationarity is attained is a function of the magnitude of ϕ_1 . The larger (smaller) the value of ϕ_1 the slower (faster) will be the speed of convergence to price equilibrium.

In order to formalize the above we recall that

$$\lim_{t\to\infty} E(P_t) = E(P^*) = \frac{\alpha-\gamma}{\delta-\beta} ,$$

and that for some specified value of t

$$\begin{aligned} Var(P_t) &= Var(\phi_{t-1}(B)w_t) \\ &= \sigma_w^2(1+\phi_1^2+\phi_2^4+\ldots+\sigma_1^{2(t-1)}) \end{aligned}$$

where σ_w^2 is the variance of w_t .

Clearly, the variance of P_t increases with t , and since $|\phi_1| < 1$ the increase in the variance of P_t for small values of t is larger than that for large values of t .

The above arguments lead us to considerations which enable us to specify a value of t , say T , at which stationarity is assumed to be achieved.

Given a $\delta > 0$, let $N(x,\delta)$ denote a <u>neighborhood</u> of x , for $x\varepsilon E_1$, where E_1 denotes the set of all real numbers. Then, given an $\varepsilon > 0$ and a $\delta > 0$, T is defined by

$$T = \sup\{t\varepsilon[0,1,2,\ldots): E(P_t)\varepsilon\ N(E(P^*),\delta) \text{ and}$$

$$(Var(P_{t+1}) - Var(P_t))\ \varepsilon\ N(0,\varepsilon)\} .$$

That is, T is the smallest value of t for which $E(P_t)$ deviates from $E(P^*)$ by less than δ and for which the increase in the variance of P_t is less than ε .

3.3.2 Speed of Convergence to Price Equilibrium.. In order to evaluate the speed with which convergence to price equilibrium is attained, we note that this is equivalent to the rate at which ϕ_1^{t-1} approaches zero for increasing values of t .

It is immediate (by inspection) that

$$\phi_1^t = 0(t^{-a})$$

for $a > 0$, and where a depends on ϕ_1 .

[Note: a function $g(t) = 0(t)$, as $t \to 0$, if $\lim_{t \to 0} \frac{g(t)}{t} = 0$] .

Thus, ϕ_1^{t-1} goes to zero faster than t^{-a} , which also goes to zero as $t \to \infty$.

3.3.3 Invertibility of the Price Process. It is well known that a finite autoregressive process is always invertible. Thus, the P_t process is always invertible. For completeness, we shall now show that the above result is also true when P_t is written in a moving average form.

The moving average process in question is invertible if $\phi_{t-1}^{-1}(B) < \infty$ for $|B| \leq 1$ [cf., Box and Jenkins (1970)]. For any specified value of t ,

$$\phi_{t-1}^{-1}(B) = \frac{1}{1 + \phi_1 B + \phi_1^2 B^2 + \ldots + \phi_1^{t-1} B^{t-1}} = \frac{1 - \phi_1 B}{1 - \phi_1^t B^t} .$$

We consider three possible situations, namely:

Case 1. $|\phi_1 B| > 1$

If $|\phi_1 B| > 1$, then $\lim_{t \to \infty} \frac{1-\phi_1 B}{1-\phi_1^t B^t} = \lim_{t \to \infty} \frac{1-\phi_1 B}{(\phi_1 B)^t} = 0$.

Case 2. $|\phi_1 B| < 1$

If $|\phi_1 B| < 1$, then $\lim_{t\to\infty} (\phi_1 B)^t = 0$.

Thus $\lim_{t\to\infty} \frac{1-\phi_1 B}{1-\phi_1^t B^t} = \lim_{t\to\infty} \frac{1-\phi_1 B}{1} = 1-\phi_1 B < 2 < \infty$.

Case 3. $|\phi_1 B| = 1$

If $|\phi_1 B| = 1$, L'Hopital's rule is required to evaluate $\lim_{t\to\infty} \frac{1-\phi_1 B}{1-\phi_1^t B^t}$.

We note $\lim_{t\to\infty} \lim_{\phi_1 B\to 1} \frac{1-\phi_1 B}{1-(\phi_1 B)^t} = \lim_{t\to\infty} \lim_{\phi_1 B\to 1} \frac{-1}{-t(\phi_1 B)^{t-1}}$

$$= \lim_{t\to\infty} + \frac{1}{t} < \infty \quad \text{for} \quad t > 0 \ .$$

Thus, $\phi_{t-1}^{-1}(B) < \infty$ is true for all values of ϕ_1 for large values of t . Thus, the invertibility conditions hold and the price series will be stationary and invertible as long as $|\beta| < |\delta|$. The relevance of invertibility will become apparent in the following section.

3.4 The Forecast Function

Our formulation and our analyses of the P_t process enable us to obtain the minimum mean square error (MMSE) forecasts of the price using its previous values. The invertibility property assures us that only a finite number of previous prices will be required for forecasts of future prices.

Let $\hat{P}_{t+k}$, $k = 1,2,\ldots$, denote the MMSE forecast of P_{t+k} . Then, using the autoregressive form of the P_t process (Equation (3.3)), it follows [cf. Box and Jenkins (1970) p. 150] that when $|\beta| < |\delta|$,

$$\hat{P}_{t+k} = P^* + (P_t - P^*)\,(\beta/\delta)^k \ .$$

Thus, the k period ahead forecast is P^*, the equilibrium price plus the weighted deviation of the current price P_t from the equilibrium price P^*, where the weight $(\beta/\delta)^k$ is a function of k. As k gets large the forecast tends to its equilibrium value P^*.

Because of the invertibility property of the P_t process we note that the forecast of future prices is a function of recent prices only. In fact, since the autoregressive representation is of order one all future forecasts are a function of the current price only.

4. An Example Motivating Our Choice of the Model

To motivate our choice of the cobweb model, we analyzed the series of annual prices of cotton in the United States from 1929 through 1971. This series is presented in Appendix A. The cobweb model was hypothesized to explain the interactions in the cotton market with the resulting price and quantity series. The error structure was assumed as discussed in Section 3.1.

4.1 Two Alternative Versions of the Cobweb Model

We considered both the original cobweb and its modification presented in Section 2 as candidate models to describe these data. P_t, the original price series, is plotted in Figure 4.1. Several important facts emerge from an examination of this plot.

First, we noted the general increase in cotton prices. This led us to allow for the possibility of a linear deterministic increase in the price series due to the effects of inflationary pressures and not as a result of the cobweb mechanism.

Second, we noted that possible stochastic elements can impact on the level of the prices, while some homogeneity exists at any given level. To account for this possible stochastic change in level, the first differences of the price series were considered for our analysis.

The above observations led us to exclude further consideration of the basic cobweb model as the possible generating mechanism of the price series. This was confirmed by our inability to empirically justify an AR(1) process for the price series.

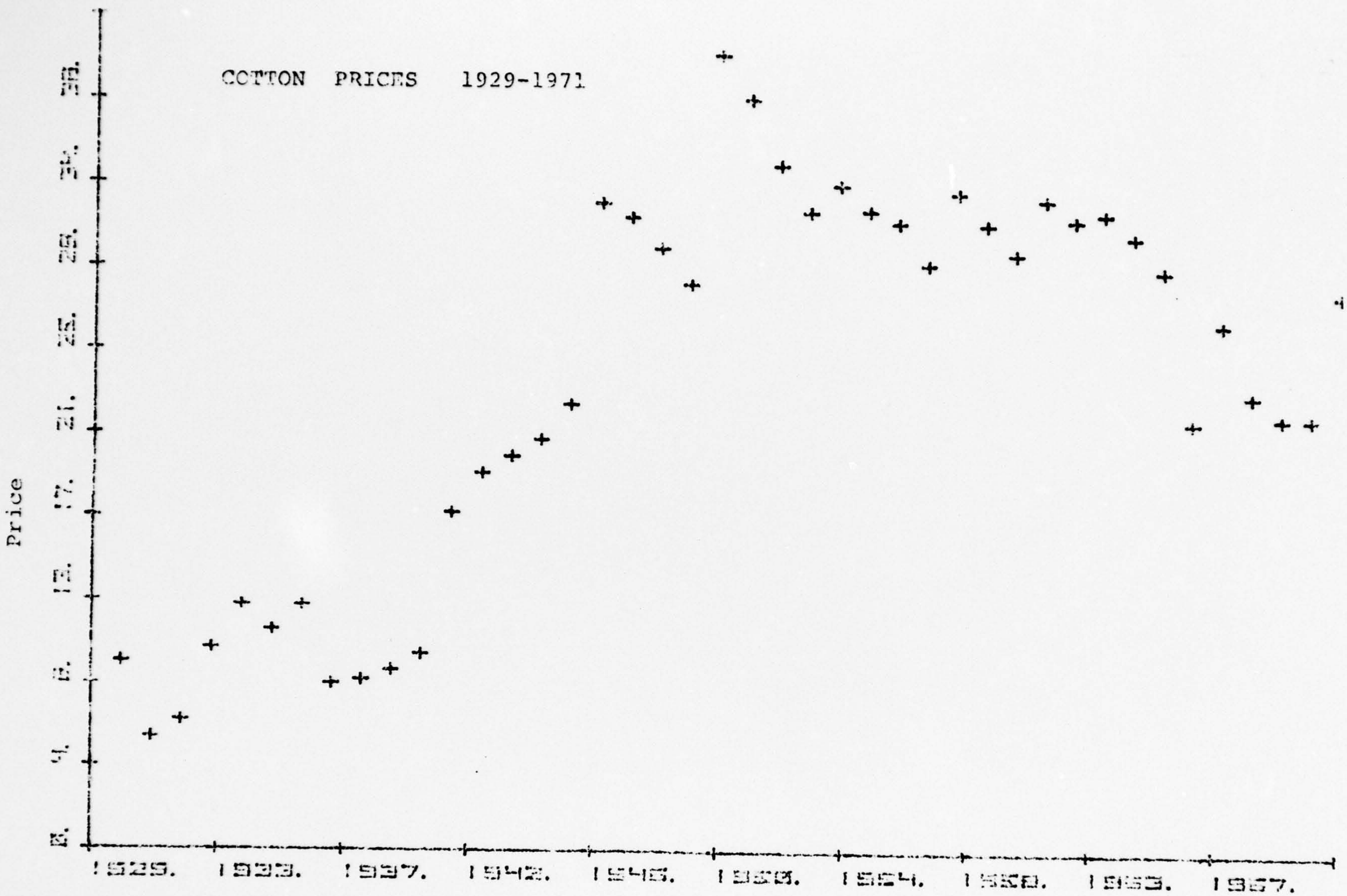
COTTON PRICES 1929-1971
Price
38.
34.
29.
25.
21.
17.
13.
8.
4.
0.
1929.
1933.
1937.
1942.
1946.
1950.
1954.
1958.
1963.
1967.

Figure 4.1

On the contrary, if the modified cobweb model is the generating mechanism of the series, then the first difference of the price should be reasonably well modelled by an AR(1) process. Accordingly, following the notation of Box and Jenkins (1970), the model

$$(1-\phi_1 B)(1-B)P_t = \theta_0 + a_t \tag{4.1}$$

was attempted on the data. This endeavor resulted in an estimate of -.0465 for ϕ_1 and 0.464 for θ_0, yielding

$$(1+0.0465B)(1-B)P_t = 0.464 + a_t \tag{4.2}$$

as an AR(1) model for $\nabla P_t = (1-B)P_t$.

In order to ascertain the adequacy of the ARIMA (1,1,0) model, the portmanteau lack of fit test [cf. Box and Pierce (1970)] was applied to the first ten autocorrelations of the residues. Based upon this test, we do not have any reason to reject the adequacy of this model at the 5% level of significance.

Thus, we can hypothesize that the cobweb model in the first differences of the price and a deterministic trend (due to inflation) is adequate as a generating mechanism of the price series. However, our analysis is no proof of the fact that the price series does in fact truly behave as Equation (4.2). Our analysis merely indicates a lack of sufficient evidence against the chosen model. At this juncture, it should also be noted that since a value of -0.0465 for ϕ_1 is not statistically significant, the possibility remains that these data can also be modelled as a random walk.

4.2 Other Possible Models

Thus far, we have discussed the adequacy of the basic cobweb, or its variant, as a possible generating mechanism of the price series. We now turn our attention to seek alternate models which in some sense may better describe our data. Such is also the spirit of Zellner and Palm (1974); properties of the dynamic simultaneous equation model are exploited in an effort to find an optimal lag size so a polynomial lag operator can be applied to the economic

variables in a system of equations. Thus, Zellner and Palm iterate in on a model which is in reasonable accord with the information in a sample of data. It is important that the resultant model have some relationship to the economic mechanism under investigation. Simple compliance with the data may yield a model with no corresponding economic rationale.

To illustrate this point, our data appear to fit an AR(2) model of the form

$$(1-\phi_1 B-\phi_2 B^2)(1-B)P_t = \theta_0 + a_t$$

better than that specified by Equation (4.1). One reason for this is the addition of an extra parameter, ϕ_2 . Estimates of ϕ_1 and ϕ_2 turned out to be -0.12 and -0.091, respectively; we note the small value of ϕ_2 . Such a model would correspond to a further modification of the cobweb of the form

$$Q_t^S = \alpha + \beta(P_{t-2}-P_{t-3}) \tag{4.3}$$

$$Q_t^D = \gamma + \delta(P_t-P_{t-1}) \tag{4.4}$$

which leads to

$$\Delta P_t = \frac{\alpha-\gamma}{\delta} + \frac{\beta}{\delta}\Delta P_{t-2} .$$

This model would correctly reflect a cobweb situation wherein a farmer needed two years to plan his crop, or for some reason such as ordering supplies, or his stubborn feeling that last year's phenomenon will affect next year's prices. He therefore makes decisions with a two-year lag. However, it appears that cotton is a crop that does not take two years to grow, so that the model (4.3)-(4.4) is somewhat suspect; thus, we would be hesitant to suggest it as an alternative to (4.1), unless some physical situation suggested its relevance. There may indeed be a two-year lag in cotton production and our analysis has pointed out its occurrence and gives us something to investigate. But until both the model and the sample data are compatible as in (4.1), we would not let the data determine the model, but rather would let the data produce grounds for further examination of the physical mechanism behind the model.

Appendix A

Cotton Prices Received by Farmers Source:
Agricultural Statistics, U.S. Department
of Agriculture, 1973

Year	Price ¢	Year	Price ¢	Year	Price ¢
1929	16.78	1944	20.73	1959	31.66
1930	9.46	1945	22.52	1960	30.19
1931	5.66	1946	32.64	1961	32.92
1932	6.52	1947	31.93	1962	31.90
1933	10.17	1948	30.38	1963	32.23
1934	12.36	1949	28.58	1964	31.07
1935	11.09	1950	40.07	1965	29.37
1936	12.36	1951	37.88	1966	21.75
1937	8.41	1952	34.59	1967	26.70
1938	8.60	1953	32.25	1968	23.11
1939	9.09	1954	33.61	1969	22.00
1940	9.89	1955	32.33	1970	21.98
1941	17.03	1956	31.75	1971	28.23
1942	19.04	1957	29.65		
1943	19.88	1958	33.23		

REFERENCES

[1] AKERMAN, GUSTAV (1957). The cobweb theorem: a reconsideration. The Quarterly Journal of Economics **71** 151-160.

[2] ANDERSON, O. D. (1975). Time Series Analysis and Forecasting: The Box-Jenkins Approach. Butterworth & Co., London.

[3] BOX, G. E. P. and G. M. JENKINS (1970). Time Series Analysis, Forecasting and Control. Holden-Day, San Francisco.

[4] BOX, G. E. P. and D. A. PIERCE (1970). Distribution of residual autocorrelations in autoregressive-integrated moving average time series models. Journal of the American Statistical Association **64** 1509-1526.

[5] COOPER, S. P. and C. R. NELSON (1975). The ex-ante prediction performance of the St. Louis and FRB-MIT-PENN econometric models and some results on composite predictors. Journal of Money, Credit, and Banking **7** 1-32.

[6] COOPER, R. L. and D. W. JORGENSON (1969). The Predictive Performance of Quarterly Econometric Models of the U.S.; Econometric Models of Cyclical Behavior. National Bureau of Economic Research, New York.

[7] NARASIMHAM, G. V. L., V. F. CASTELLINO and N. D. SINGPURWALLA (1974). On the predictive performance of the BEA quarterly econometric model and a Box-Jenkins type ARIMA model. Business and Economic Statistics, Section 1974, Proceedings of the American Statistical Association, Washington, D.C.

[8] NELSON, CHARLES R. (1973). Applied Time Series Analysis for Managerial Forecasting. Holden-Day, San Francisco.

[9] NERLOVE, MARC (1958). Adaptive expectations and cobweb phenomena. The Quarterly Journal of Economics **73** 227-40.

[10] NICHOLSON, WALTER (1972). Microeconomic Theory, Basic Principles and Extensions. The Dryden Press, Inc., Hinsdale, Illinois.

[11] RADER, TROUT (1972). Theory of Microeconomics. Academic Press, New York.

[12] U. S. Department of Agriculture (1973). Agricultural Statistics 1973. Washington, D.C.

[13] ZELLNER, A. and F. PALM (1974). Time series analysis and simultaneous equation econometric models. Journal of Econometrics **2** 17-54.

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